Correct-by-Design Control Synthesis for Multilevel Converters using State Space Decomposition

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¹LSV, ²SATIE - ENS Cachan & CNRS

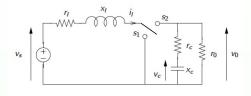
FSFMA, 13 May 2014

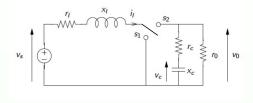
Plan

- 1 Context: Multilevel Converters
- 2 Controllability of Switched Systems
- 3 State Decomposition
- 4 Application to Multilevel Converters

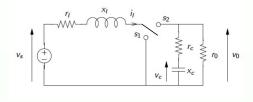
Context: Multilevel Converters



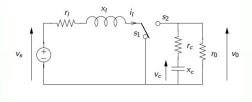




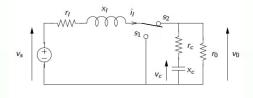
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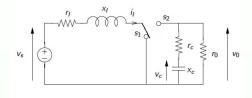
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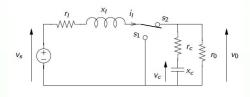
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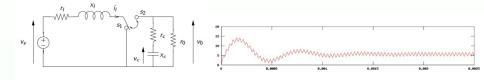
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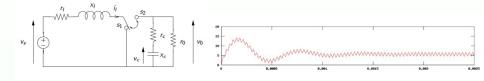
$$\dot{x} = f_2(x) = \begin{pmatrix} -\frac{1}{x_l} (r_l + \frac{r_0 \cdot r_c}{r_0 + r_c}) & -\frac{1}{x_l} \frac{r_0}{r_0 + r_c} \\ \frac{1}{x_c} \frac{r_0}{r_0 + r_c} & -\frac{1}{x_c} \frac{1}{r_0 + r_c} \end{pmatrix} x + \begin{pmatrix} \frac{v_s}{x_l} \\ 0 \end{pmatrix}$$

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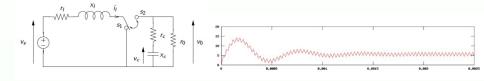
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NB: widespread in portable electronic devices (phones, laptops) supplied with batteries, which contain sub-circuits, each with its own voltage level requirement \neq from that supplied by the battery

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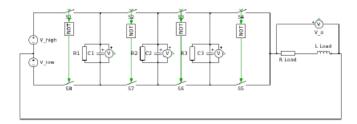
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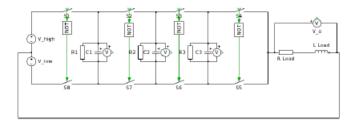
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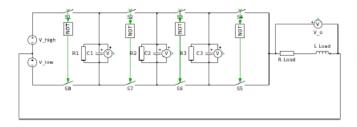
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- Other advantage: can operate at low switching frequency (~ lower switching loss and stress, higher efficiency)

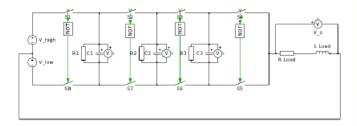




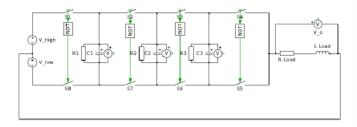
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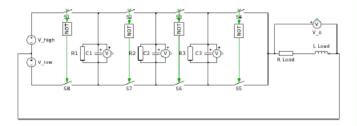
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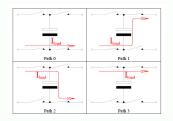


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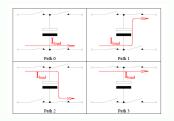
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- \blacksquare \sim transform a DC voltage into a staircase waveform (\approx sinusoidal)

Focus

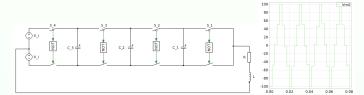


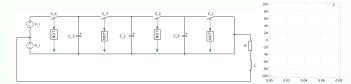
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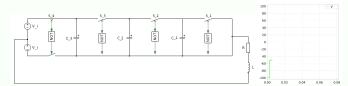
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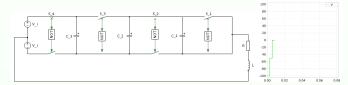


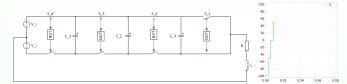
- According to the position of S_i and S_{i+1} , the capacitor C_i contributes or not to the output voltage.
- By global positioning of the switching cells, one is thus able to fraction the output voltage between $-v_i$ and $+v_i$.

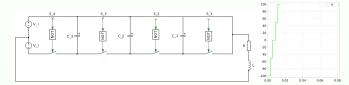


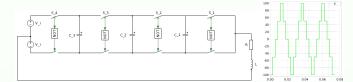










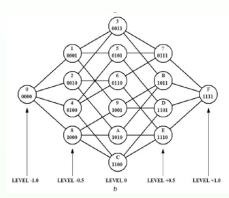


Example: 5-level multilevel converter

■ A priori, many switching sequences exist: paths of a graph where nodes correspond to modes $(S_1S_2S_3S_4)$ with $S_i = 0, 1$, and adjacent nodes differ by one bit

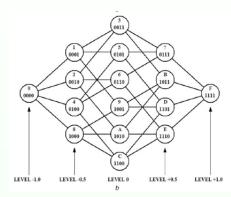
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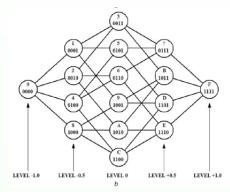
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- for n cycles: $(576)^n$ paths



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- In industry, use of heuristic rules in order to find predefined sequences of patterns and apply them repeatedly (no formal guarantee of capacitor voltage balance property)

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The problem is to synthesize a safety controller for the switching system (correct-by-design controller)

Controllability of Switched Systems

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- duration of cycle $T = 8\tau$
- The 5-level converter can be seen as a switched system. Given a mode S, the associated dynamics is of the form $\dot{x}(t) = A_S x(t) + b_S$ with:

$$A_{S} = \begin{pmatrix} -\frac{1}{R_{1}C_{1}} & 0 & 0 & \frac{S_{1}-S_{2}}{C_{1}} \\ 0 & -\frac{1}{R_{2}C_{2}} & 0 & \frac{S_{2}-S_{3}}{C_{2}} \\ 0 & 0 & -\frac{1}{R_{3}C_{3}} & \frac{S_{3}-S_{4}}{C_{3}} \\ \frac{S_{2}-S_{1}}{L_{Load}} & \frac{S_{3}-S_{2}}{L_{Load}} & \frac{S_{4}-S_{3}}{L_{Load}} & -\frac{R_{Load}}{L_{Load}} \end{pmatrix} b_{S} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{(2S_{1}-1)v_{input}}{L_{Load}} \end{pmatrix}$$

Safety Control Problem

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■ If a fluctuation of $\pm 5V$ is admissible, the safety area is: $R = [145, 155] \times [95, 105] \times [45, 55]$

State Decomposition

 \blacksquare Given a safety area R

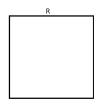
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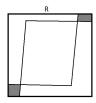
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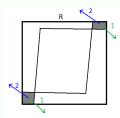
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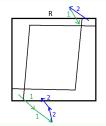
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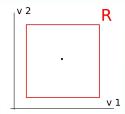
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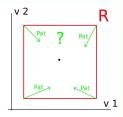
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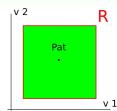
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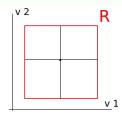
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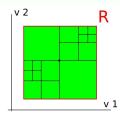
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- If such a pattern exists, then uniform control over the whole R



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- In case of failure, iterate the bisection



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Given a set R, find by iterated bisection a decomposition Δ , i.e., a set of couples $\{(V_i, \pi_i)\}_{i \in I}$ such that:

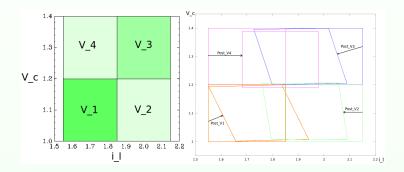
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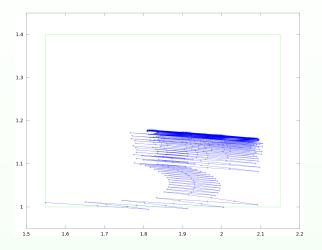
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NB: The basic procedure can be refined with an extended safety set R^* in order to guarantee that all the intermediate points are in R^* at each sampling time.

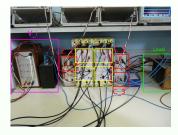
Trajectory under the induced control



Application to Multilevel Converters

Case Study: a Multilevel Converter

■ A prototype built by SATIE Lab for the Farman project BOOST2



■ The general function of a multilvel converter is to synthesize a desired AC voltage from several levels of DC voltages.

Application to Multilevel Converter (5 levels)

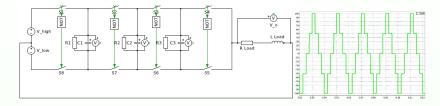


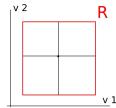
Figure : Electric scheme and ideal output for 5-level converter

Objective:

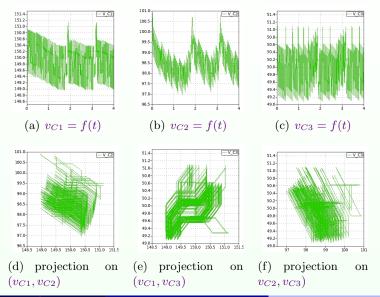
Find appropriate switching strategy in order to obtain the desired staircase output voltage while keeping capacitor voltage balancing

Application of Decomposition Procedure

- $v_{input} = 100V, R_{load} = 50\Omega, C_1 = C_2 = C_3 = 0.0012F,$ $L_{load} = 0.2H, R_1 = R_2 = R_3 = 20,000\Omega, T = 8\tau = 0.02s$ (frequency 50Hz)
- $R = [145, 155] \times [95, 105] \times [45, 55]$
 - procedure successful using only one bisection by dimension



Numerical Simulations of the Capacitor Voltages



Numerical Simulation of the Output Voltage



Physical Experimentation on the Prototype

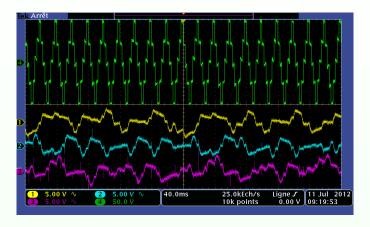


Figure: Output voltage and capacitor voltages

Correct-by-design control!

Other Results

- preliminary results about robustness on the prototype
- positive results also for multilevel with 7 levels
- prototype MINIMATOR available at https://bitbucket.org/ukuehne/minimator

■ New formal method for synthesis of correct-by-design control for safety properties

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- Decomposes the safety set into large zones of uniform control
- Based on forward computation (\sim better numerical stability for contractive systems)
- State-dependent control
- Successfully applied to physical prototype built by electrical engineering SATIE laboratory

Perspectives

- Improvement of the scalability of the approach (parallelization of the computations, e.g., GPU...)
- Refinement of the bisection method
- Addition of optimisation objectives
- Further work on robustness of the method in presence of variable resistive load.

Thanks!

Thanks!

Questions?