The $1/N$ Expansion in Colored Tensor Models

Răzvan Gurău

Laboratoire d'Informatique de Paris-Nord, 2011

Matrix Models

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A success story: Matrix Models in two dimensions

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All these applications rely crucially on the "$1/N$" expansion!

Ribbon Graphs as Feynman Graphs

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Consider the partition function.

$$Z(Q) = \int [d\phi] \; e^{-N\left(\frac{1}{2} \sum \phi_{a_1 a_2} \delta_{a_1 b_1} \delta_{a_2 b_2} \phi^*_{b_1 b_2} + \lambda \sum \phi_{a_1 a_2} \phi_{a_2 a_3} \phi_{a_3 a_1} \right)}$$

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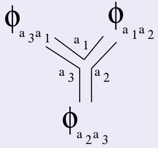
The vertex is a ribbon vertex because the field ϕ has two arguments.



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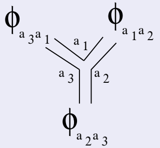

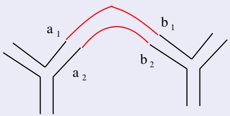

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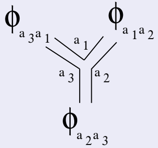
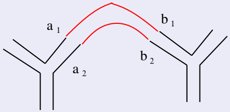
The vertex is a ribbon vertex because the field ϕ has two arguments. The lines conserve the two arguments (thus having two strands). The strands close into faces.





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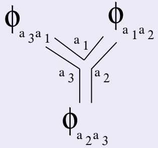

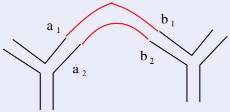

$Z(Q)$ is a sum over ribbon Feynman graphs.

Amplitude of Ribbon Graphs

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The Amplitude of a graph with $\mathcal{N}$ vertices is

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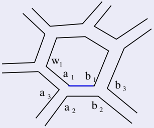

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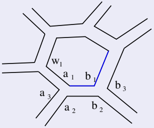

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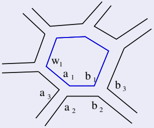

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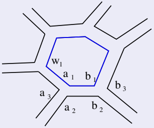

$$\sum \delta_{a_1 b_1} \delta_{b_1 c_1} \dots \delta_{w_1 a_1} = \sum \delta_{a_1 a_1} = N$$

$$A = \lambda^{\mathcal{N}} N^{\mathcal{N}-\mathcal{L}+\mathcal{F}} = \lambda^{\mathcal{N}} N^{2-2g(\mathcal{G})}$$

with $g_{\mathcal{G}}$ is the genus of the graph.

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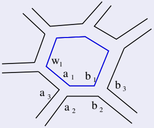

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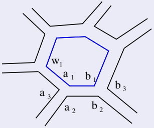

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Ribbon Graphs are Dual to Discrete Surfaces

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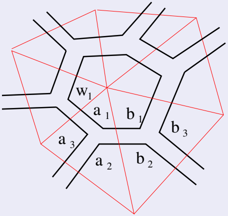

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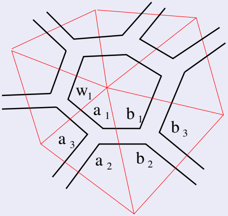

Place a point in the middle of each face.

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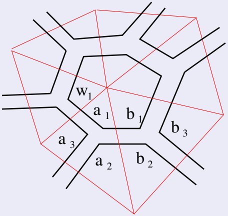

Place a point in the middle of each face. Draw a line crossing each ribbon line.

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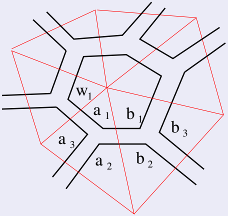

Place a point in the middle of each face. Draw a line crossing each ribbon line. The ribbon vertices correspond to triangles.

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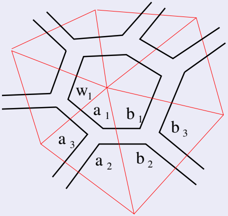

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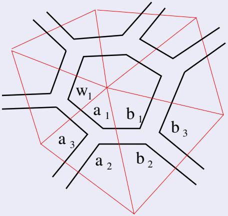

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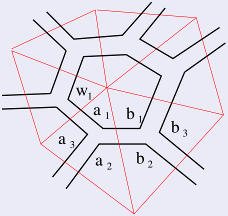

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Matrix models sum over all graphs (i.e. surfaces) with canonical weights (Feynman rules). The dominant planar graphs represent spheres.

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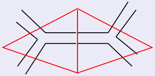
surfaces $\leftrightarrow$ ribbon graphs



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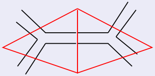
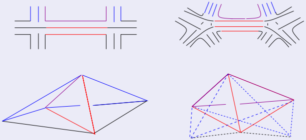


D dimensional spaces $\leftrightarrow$ colored stranded graphs



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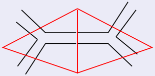

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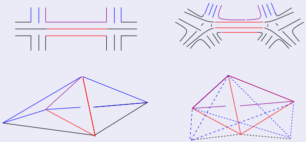

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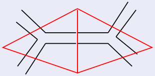

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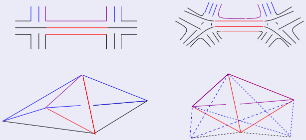

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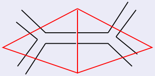

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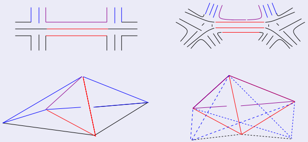

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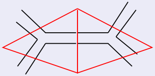

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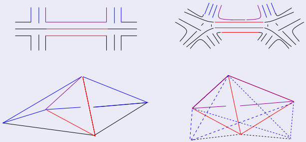

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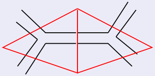

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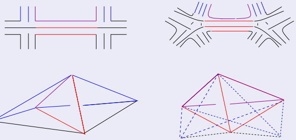

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$1/N$ expansion in the genus
$A(\mathcal{G}) = N^{2-2g(\mathcal{G})}$

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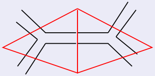

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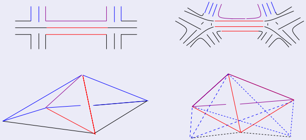

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$1/N$ expansion in the genus
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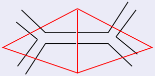
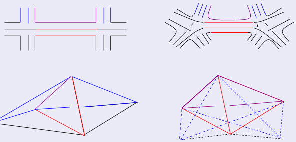
From Matrix to COLORED Tensor Models

surfaces $\leftrightarrow$ ribbon graphs



D dimensional spaces $\leftrightarrow$ colored stranded graphs



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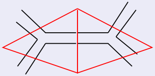

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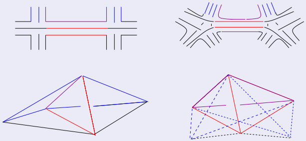

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Introduction

Colored Tensor Models

- Colored Graphs
- Jackets and the $1/N$ expansion
- Topology
- Leading order graphs are spheres

Conclusion

Colored Stranded Graphs

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Clockwise and anticlockwise turning colored vertices (positive and negative oriented D simplices).

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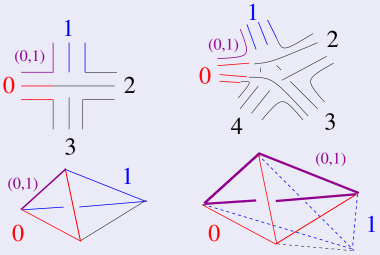

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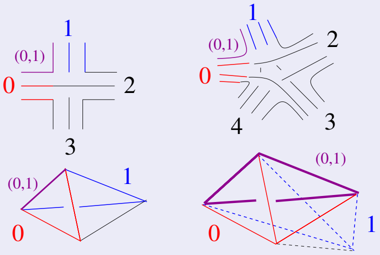

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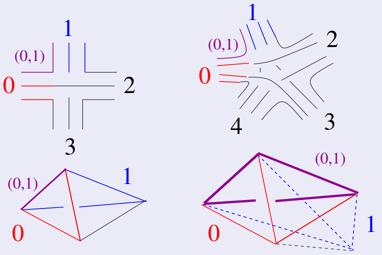

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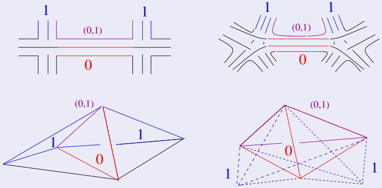

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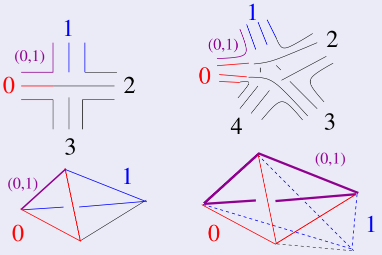

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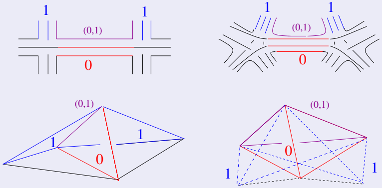

Strands are identified by a couple of colors ($D-2$ simplices).

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Compute $\mathcal{F}$!

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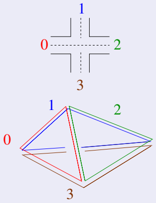

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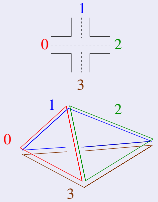

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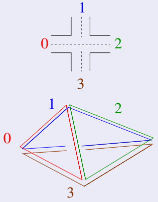

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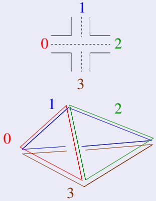

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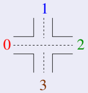

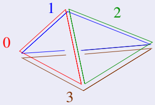

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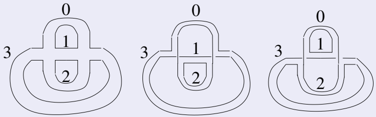

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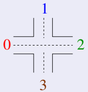

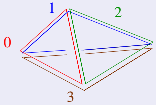

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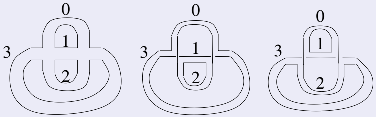

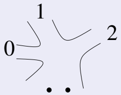

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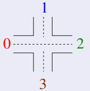

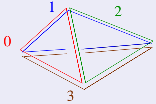

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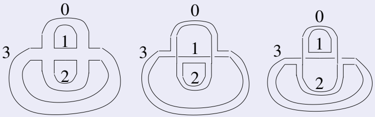

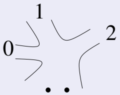

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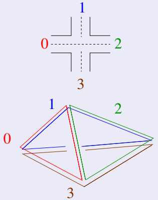

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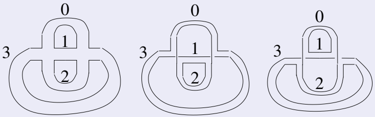

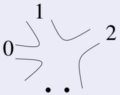

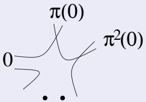

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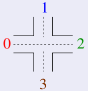

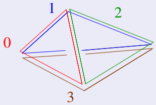

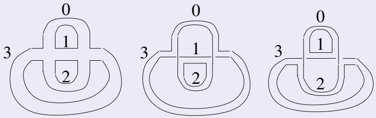

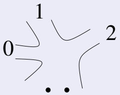

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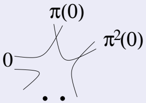

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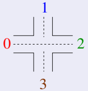

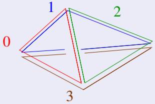

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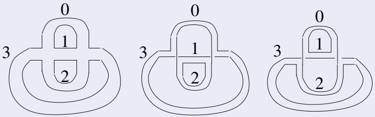

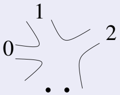

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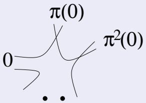

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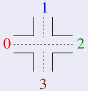

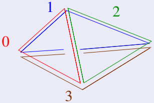

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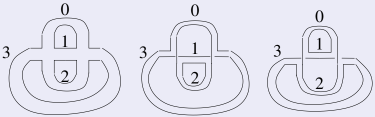

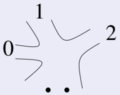

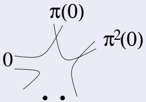

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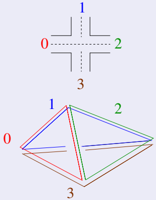

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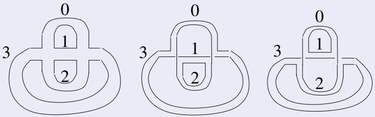

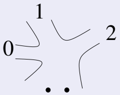

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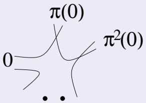

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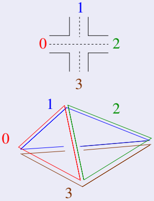

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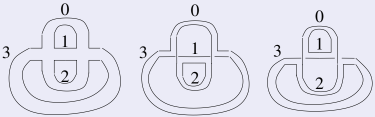

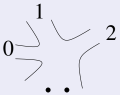

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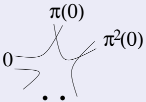

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The amplitude of a graph is given by its degree

$$A^{\mathcal{G}} = (\lambda\bar{\lambda})^p \; N^{-p\frac{D(D-1)}{2} + \mathcal{F}} = (\lambda\bar{\lambda})^p \; N^{D - \frac{2}{(D-1)!}\omega(\mathcal{G})}$$

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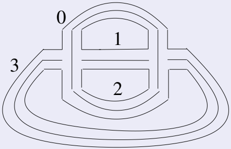

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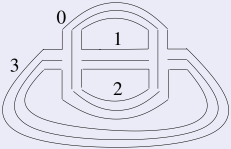

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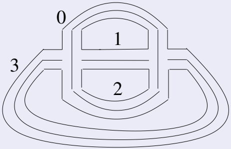

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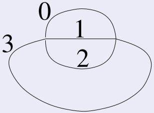

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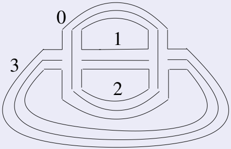

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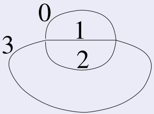

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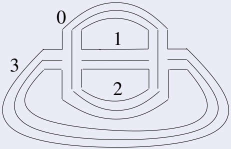

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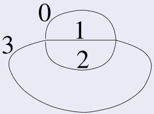

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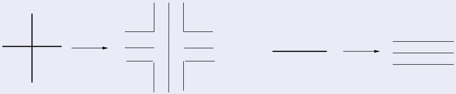

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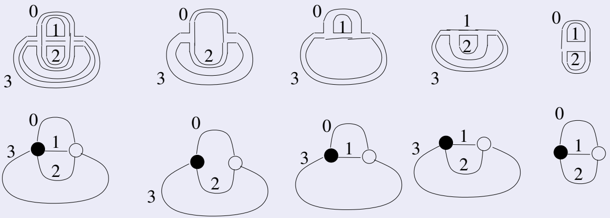

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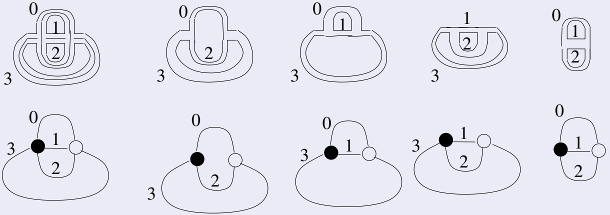

A colored graph $\mathcal{G}$ is dual to an orientable, normal, D dimensional, simplicial pseudo manifold. Its n-bubbles are dual to the links of the $D-n$ simplices of the pseudo manifold.

Topology 3: Homeomorphisms and 1-Dipoles

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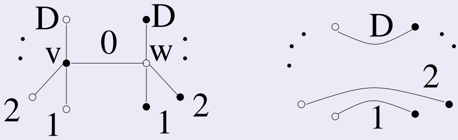

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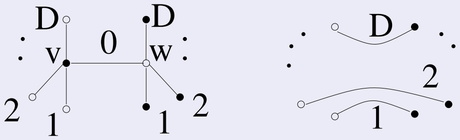

A 1-dipole: a line (say of color 0) connecting two vertices $v \in \mathcal{B}^{1\dots D}_{(\alpha)}$ and $w \in \mathcal{B}^{1\dots D}_{(\beta)}$ with $\mathcal{B}^{1\dots D}_{(\alpha)} \neq \mathcal{B}^{1\dots D}_{(\beta)}$.

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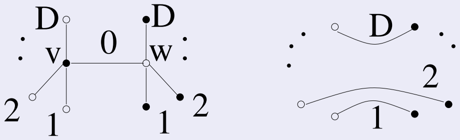

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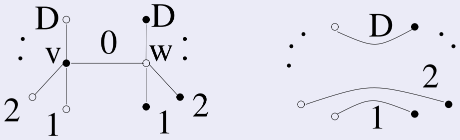

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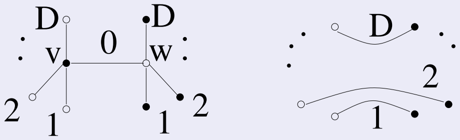

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It is in principle very difficult to check if a bubble is a sphere or not.

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Degree 0 Graphs are Spheres

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- Generalize the results obtained using matrix models in higher dimensions.