Regulated Grammars and Automata

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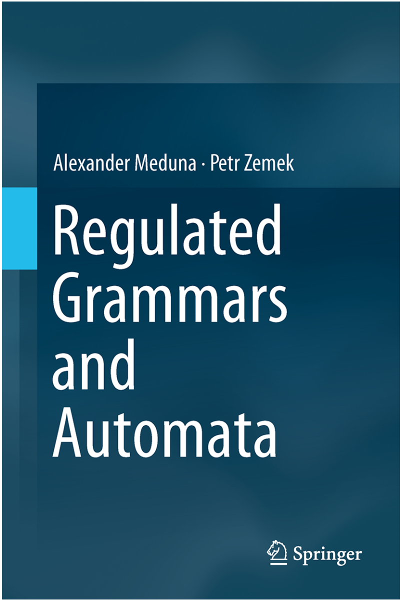
Book Cover


Alexander Meduna · Petr Zemek
Regulated Grammars and Automata
Springer

Outline

- **Part I: An Introduction to the Book**
  - Basic Idea
  - General Info
  - Contents
- **Part II: A Sample: One-Sided Random Context Grammars**
  - Basic Concept
  - Definitions and Examples
  - Generative Power
  - Normal Forms
  - Reduction
  - Applications

- a grammar or an automaton based upon a finite set of rules R

Example

A context-free grammar with the set of rules R:

$$R: \quad \begin{aligned} S &\rightarrow ABC \\ A &\rightarrow aA \\ B &\rightarrow bB \\ C &\rightarrow cC \\ A &\rightarrow a \\ B &\rightarrow b \\ C &\rightarrow c \end{aligned}$$

- a grammar or an automaton based upon a finite set of rules R
- a regulation over R

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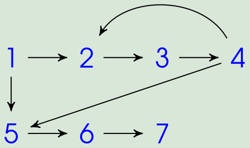
& 7: C \rightarrow c
\end{aligned}
$$



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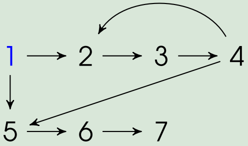
\end{aligned}
$$

$$S \Rightarrow ABC \qquad [1]$$



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\begin{aligned}
S & \Rightarrow ABC & [1] \\
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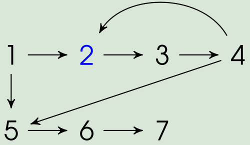

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\begin{array}{rrlcl}
R: & 1: & S & \rightarrow & ABC \\
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\end{array}
$$

$$
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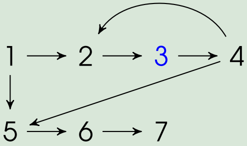
 & \Rightarrow & aAbBC & [\color{blue}{3}]
\end{array}
$$



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\end{aligned}
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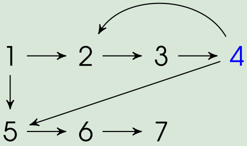

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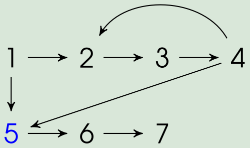

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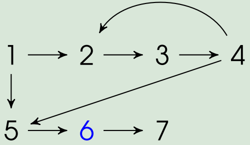

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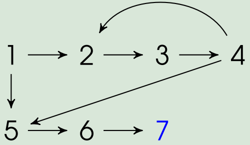

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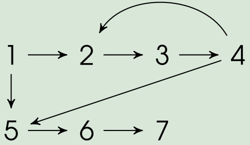

$$L(G) = \{a^n b^n c^n : n \geq 1\}$$

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Motivation and Subject

- an important trend in formal language theory
- since 1990, no book has been published on the subject although many papers have discussed it

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Motivation and Subject

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Purpose

- theoretical: to summarize key results on the subject
- practical: to demonstrate applications of regulated grammars and automata

General Info (Continued)

Focus

- power
- transformation
- reduction

Focus

- power
- transformation
- reduction

Organization

- 9 parts
- 22 chapters

Approach and Features

- theoretically oriented treatment of regulated grammars and automata
- emphasis on algorithms
- intuitive explanation
- many examples
- application perspectives

General Info (Continued)

Approach and Features

- theoretically oriented treatment of regulated grammars and automata
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- application perspectives

Book Audience

- computer scientists: professionals, professors, Ph.D. students
- mathematicians
- linguists

Contents (1/4)

Contents (1/4)

Contents (2/4)

Part III Regulated Grammars: Special Topics

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Contents (3/4)

Part V Regulated Grammar Systems

Contents (3/4)

Contents (3/4)

Contents (4/4)

Part VIII Applications

Part II: A Sample:
One-Sided Random Context Grammars

- a modification of context-free grammars
- $(A \rightarrow x, U, W) \in P$

Random Context Grammars: Basic Concept

- a modification of context-free grammars
- $(A \rightarrow x, U, W) \in P$

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$$\longleftarrow\cdots\cdots \boxed{A} \cdots\cdots\longrightarrow$$

Illustration

$(A \rightarrow x, \{B, C\}, \{D\}) \in P$

$bBcECbAcB$

Random Context Grammars: Basic Concept

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$(A \rightarrow x, \{B, C\}, \{D\}) \in P$

$$\overleftarrow{bBcECb}\ \boxed{A}\ \overrightarrow{cB} \Rightarrow bBcECb\,x\,cB$$

One-Sided RCGs: Basic Concept

- a variant of random context grammars
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$$\ldots\ldots \boxed{A} \xrightarrow{\ldots\ldots}$$

Illustration

$(A \rightarrow x, \{B, C\}, \{D\}) \in P_L$

$$bBcECbAcD$$

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Illustration

$(A \rightarrow x, \{B, C\}, \{D\}) \in P_L$

$$\overleftarrow{bBcECb} \boxed{A} cD \Rightarrow bBcECb\, x\, cD$$

Definition

A *one-sided random context grammar* is a quintuple

$$G = (N, T, P_L, P_R, S)$$

where

- N is an alphabet of *nonterminals*;
- T is an alphabet of *terminals* ($N \cap T = \emptyset$);
- P_L and P_R are two finite sets of *rules* of the form

  $$(A \rightarrow x, U, W)$$

  where $A \in N$, $x \in (N \cup T)^*$, and $U, W \subseteq N$;
- $S \in N$ is the *starting nonterminal*.

Definitions

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Definition

If $(A \rightarrow x, U, W) \in P_L \cup P_R$ implies that $|x| \geq 1$, then G is *propagating*.

Definition

The *direct derivation* $\Rightarrow$ is defined as

$$uAv \Rightarrow uxv$$

if and only if

$$(A \rightarrow x, U, W) \in P_L, U \subseteq \text{alph}(u), \text{ and } W \cap \text{alph}(u) = \emptyset$$

or

$$(A \rightarrow x, U, W) \in P_R, U \subseteq \text{alph}(v), \text{ and } W \cap \text{alph}(v) = \emptyset$$

Note: alph(y) denotes the set of all symbols appearing in string y

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Note: alph(y) denotes the set of all symbols appearing in string y

Definition

The *language of G* is defined as

$$L(G) = \{w \in T^* : S \Rightarrow^* w\}$$

where $\Rightarrow^*$ is the reflexive-transitive closure of $\Rightarrow$.

Example

Consider the one-sided random context grammar

$$G = (\{S, A, B, \bar{A}, \bar{B}\}, \{a, b, c\}, P_L, P_R, S)$$

where P_L contains

$$(S \rightarrow AB, \emptyset, \emptyset) \qquad (\bar{B} \rightarrow B, \{A\}, \emptyset)$$
$$(B \rightarrow b\bar{B}c, \{\bar{A}\}, \emptyset) \qquad (B \rightarrow \varepsilon, \emptyset, \{A, \bar{A}\})$$

and P_R contains

$$(A \rightarrow a\bar{A}, \{B\}, \emptyset) \qquad (A \rightarrow \varepsilon, \{B\}, \emptyset)$$
$$(\bar{A} \rightarrow A, \{\bar{B}\}, \emptyset)$$

Example

P_L: $(S \to AB, \emptyset, \emptyset)$
$(B \to b\bar{B}c, \{\bar{A}\}, \emptyset)$
$(\bar{B} \to B, \{A\}, \emptyset)$
$(B \to \varepsilon, \emptyset, \{A, \bar{A}\})$

P_R: $(A \to a\bar{A}, \{B\}, \emptyset)$
$(\bar{A} \to A, \{\bar{B}\}, \emptyset)$
$(A \to \varepsilon, \{B\}, \emptyset)$

$$S \Rightarrow AB \quad [(S \to AB, \emptyset, \emptyset)]$$

Example

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$$
\begin{aligned}
S &\Rightarrow AB && [(S \rightarrow AB, \emptyset, \emptyset)] \\
&\Rightarrow a\bar{A}B && [(A \rightarrow a\bar{A}, \{B\}, \emptyset)]
\end{aligned}
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Example (Continued)

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&\Rightarrow a^nb^nc^n & [(B \rightarrow \varepsilon, \emptyset, \{A, \bar{A}\})]
\end{aligned}
$$

Example

$$P_L: \quad (S \to AB, \emptyset, \emptyset) \qquad P_R: \quad (A \to a\bar{A}, \{B\}, \emptyset)$$

$$(B \to b\bar{B}c, \{\bar{A}\}, \emptyset) \qquad (\bar{A} \to A, \{\bar{B}\}, \emptyset)$$

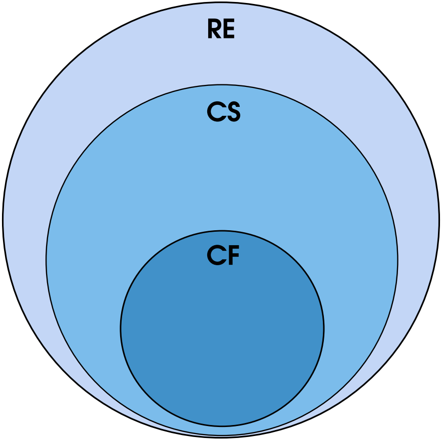
$$(\bar{B} \to B, \{A\}, \emptyset) \qquad (A \to \varepsilon, \{B\}, \emptyset)$$

$$(B \to \varepsilon, \emptyset, \{A, \bar{A}\})$$

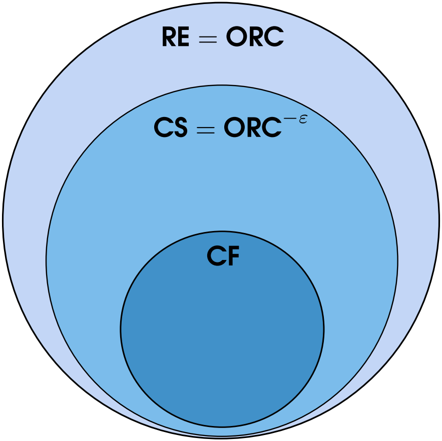
$$\begin{array}{llll}
S & \Rightarrow & AB & [(S \to AB, \emptyset, \emptyset)] \\
 & \Rightarrow & a\bar{A}B & [(A \to a\bar{A}, \{B\}, \emptyset)] \\
 & \Rightarrow & a\bar{A}b\bar{B}c & [(B \to b\bar{B}c, \{\bar{A}\}, \emptyset)] \\
 & \Rightarrow & aAb\bar{B}c & [(\bar{A} \to A, \{\bar{B}\}, \emptyset)] \\
 & \Rightarrow & aAbBc & [(\bar{B} \to B, \{\bar{A}\}, \emptyset)] \\
 & \Rightarrow^* & a^nAb^nBc^n & \\
 & \Rightarrow & a^nb^nBc^n & [(A \to \varepsilon, \{B\}, \emptyset)] \\
 & \Rightarrow & a^nb^nc^n & [(B \to \varepsilon, \emptyset, \{A, \bar{A}\})]
\end{array}$$

$$L(G) = \{a^nb^nc^n : n \geq 0\}$$

Generative Power

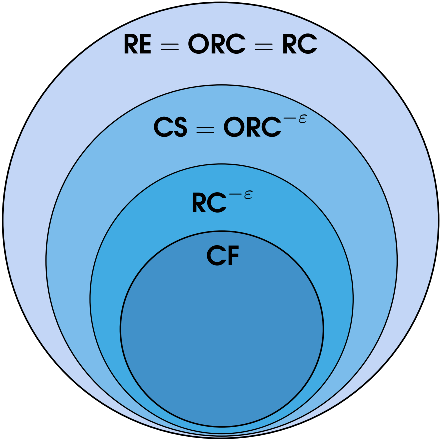


RE the family of recursively enumerable languages
CS the family of context-sensitive languages
CF the family of context-free languages



ORC the language family generated by one-sided random context grammars

$\mathbf{ORC}^{-\varepsilon}$ the language family generated by propagating one-sided random context grammars



RC the language family generated by random context grammars

RC$^{-\varepsilon}$ the language family generated by propagating random context grammars

Normal Forms

Results Structure

For any one-sided random context grammar, there is an equivalent one-sided random context grammar satisfying one of the following normal forms.

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$P_L \cap P_R = \emptyset$

Normal Form III

$(A \rightarrow x, U, W) \in P_L \cup P_R$ implies that $x \in NN \cup T \cup \{\varepsilon\}$

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$(A \rightarrow x, U, W) \in P_L \cup P_R$ implies that $x \in NN \cup T \cup \{\varepsilon\}$

Normal Form IV

$(A \rightarrow x, U, W) \in P_L \cup P_R$ implies that $U = \emptyset$ or $W = \emptyset$

- with respect to the total number of nonterminals

Theorem

Any one-sided random context grammar can be converted to an equivalent one having no more than 10 nonterminals.

- with respect to the total number of nonterminals

Theorem

Any one-sided random context grammar can be converted to an equivalent one having no more than 10 nonterminals.

- with respect to the number of right random context nonterminals

Definition

If $(A \rightarrow x, U, W) \in P_R$, then A is a *right random context nonterminal*.

Reduction

- with respect to the total number of nonterminals

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Any one-sided random context grammar can be converted to an equivalent one having no more than 10 nonterminals.

- with respect to the number of right random context nonterminals

Definition

If $(A \rightarrow x, U, W) \in P_R$, then A is a *right random context nonterminal*.

Theorem

Any one-sided random context grammar can be converted to an equivalent one having no more than 2 right random context nonterminals.

- with respect to the number of right random context rules

Definition

If $p \in P_R$, then p is a *right random context rule*.

- with respect to the number of right random context rules

Definition

If $p \in P_R$, then p is a *right random context rule*.

Theorem

Any one-sided random context grammar can be converted to an equivalent one having no more than 2 right random context rules.

General Application Area

- information processing based on the existence or absence of some information parts

Specific Scientific Disciplines

- genetics: modification of genetic codes in which some prescribed sequences of nucleotides occur while some others do not
- linguistics: generation or verification of texts that contain no forbidding passages, such as vulgarisms or classified information
- computer science: syntax analysis of complicated non-context-free structures during language translation

Discussion