On Certain Statistics of Random Weighted Partitions of Large Integers

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April 6, 2011

Abstract

A weighted partition of the positive integer n is a multiset of size n whose decomposition into a union of disjoint components (parts) satisfies the following condition: for a given sequence of non-negative numbers $\{b_k\}_{k\geq 1}$, a part of size k appears in exactly one of b_k possible types. Assuming that a weighted partition of n is selected uniformly at random from the set of all such partitions, we study the limiting distributions of the largest part size X_n and of the number of parts ξ_n as $n \to \infty$. Under certain fairly general assumptions on the Dirichlet generating series $D(s) = \sum_{k=1}^{\infty} b_k k^{-s}, s = \sigma + iy$, G. Meinardus, Math. Z. 59(1954), 388-398, has obtained the asymptotic of the total number of weighted partitions of n. We assume that Meinardus conditions hold and prove that X_n and ξ_n, appropriately normalized, converge weakly to non-degenerate probability distributions.